

Instrumental variables I

Session 11

PMP 8521: Program evaluation
Andrew Young School of Policy Studies

Plan for today

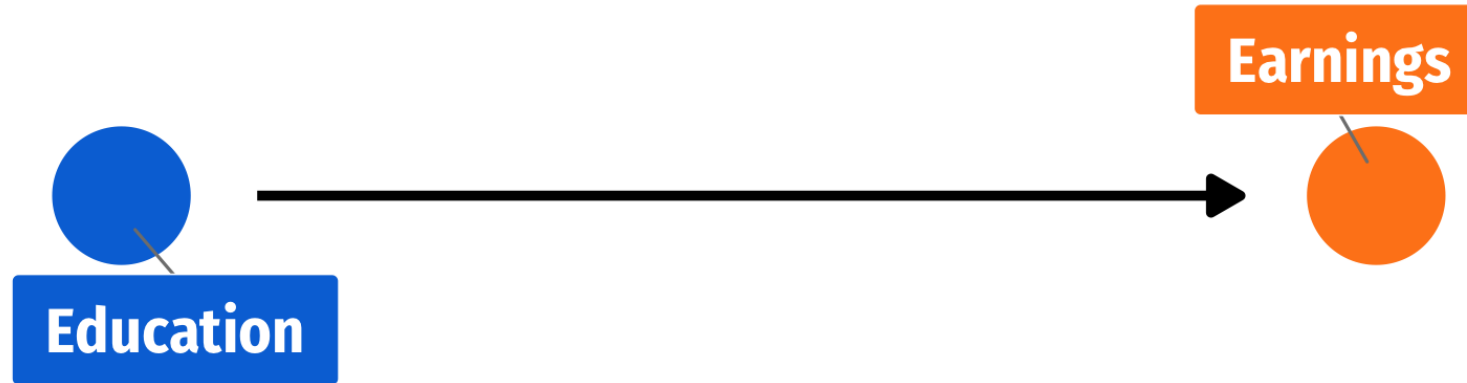
Endogeneity and exogeneity

Instruments

Using instruments

Endogeneity and exogeneity

Does education cause higher earnings?



$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

If we ran this regression, would β_1 give us the causal effect of education?

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

No!

Omitted variable bias!

Unclosed backdoors!

Endogeneity!

Exogeneity and endogeneity

Exogenous variables

Value is not determined by anything else in the model

In a DAG, a node that doesn't have arrows coming into it

Exogeneity

Education is exogenous: no arrows *into* it

Exogeneity and endogeneity

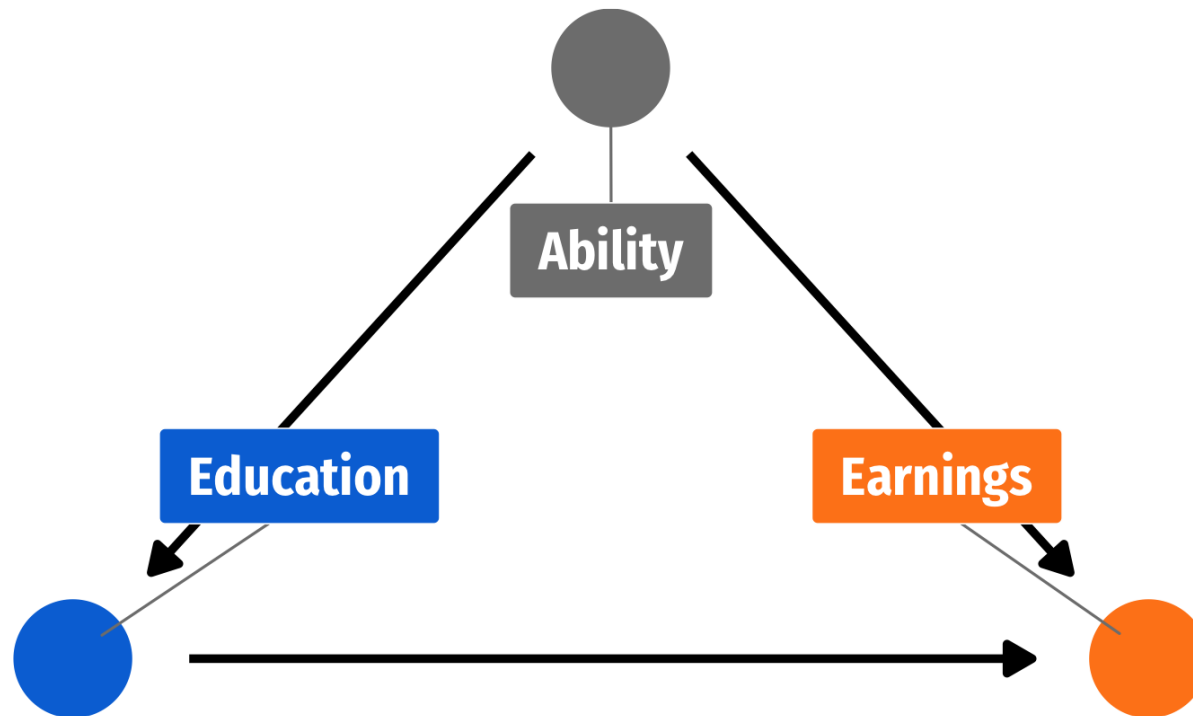
Endogenous variables

Value is determined by something else in the model

In a DAG, a node that has arrows coming into it

Endogeneity

Education is endogenous: Ability \rightarrow Education

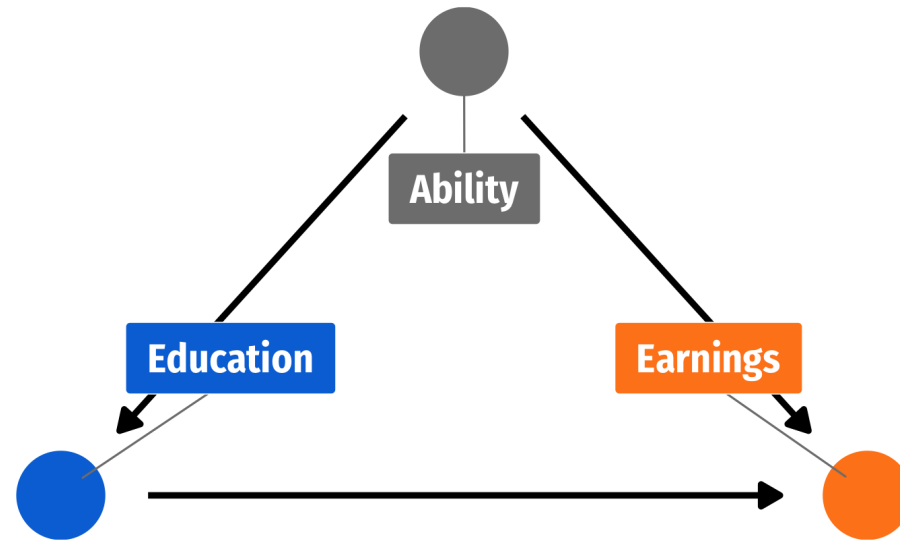


Exogeneity

What would exogenous variation in education look like?

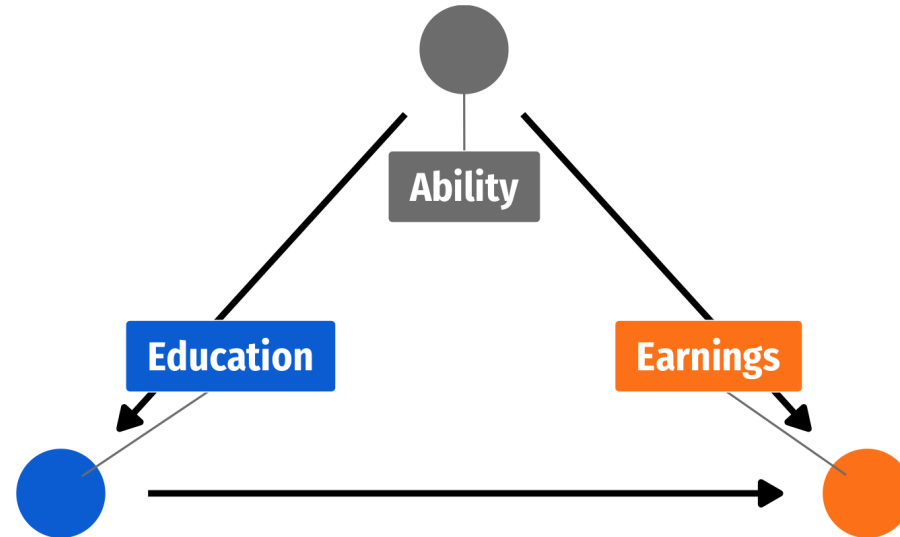
Choices to get more education that are essentially random (or at least uncorrelated with omitted variables)

**We'd like education to be exogenous
(an outside decision or intervention), but it's not!**



**Part of it is exogenous, but part of it is
caused by ability, which is in the DAG**

Fixing endogeneity with DAGs



Close backdoor and adjust for ability

Adjustment filters out the endogenous part of education and leaves us with just the endogenous part

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \beta_2 \text{Ability}_i + \varepsilon_i$$

| | Outcome = wage | |
|--------------------------------------|-----------------------|-----------------|
| | Unadjusted | Adjusted |
| (Intercept) | -59.378*** | -85.571*** |
| | (10.376) | (7.198) |
| educ | 13.124*** | 7.767*** |
| | (0.618) | (0.456) |
| ability | | 0.344*** |
| | | (0.010) |
| Num.Obs. | 1000 | 1000 |
| R2 | 0.311 | 0.673 |
| F | 451.244 | 1025.794 |
| * p < 0.1, ** p < 0.05, *** p < 0.01 | | |

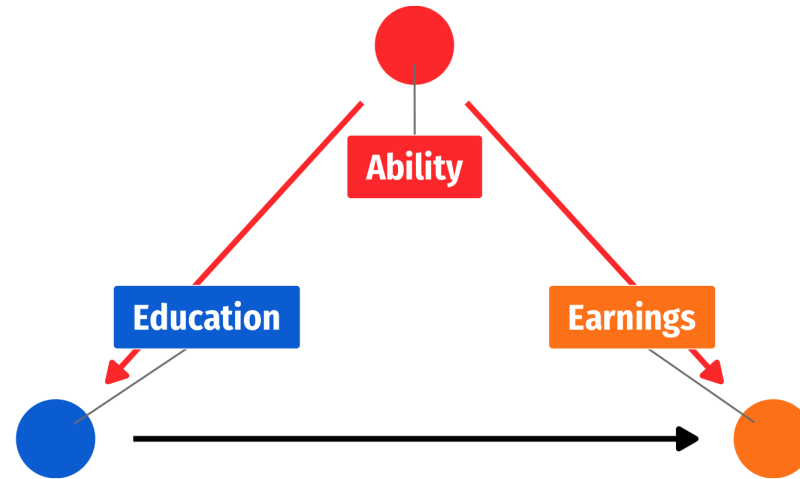
**Unadjusted
is wrong!**

**Adjusted
is right!**

**One year of education
causes hourly wage to
increase by \$7.77**

(FAKE DATA)

But we can't measure ability!



$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \beta_2 \text{Ability}_i + \varepsilon_i$$

Unmeasurable ability node is in the error term (ε)

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

Split exogeneity and endogeneity

What if we could somehow separate education into its endogenous and exogenous parts?

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

$$\beta_0 + \beta_1 (\text{Education}_i^{\text{exog.}} + \text{Education}_i^{\text{endog.}}) + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \underbrace{\beta_1 \text{Education}_i^{\text{endog.}}}_{\omega_i} + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$

Find exogeneity with One Weird Trick™

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$

How do we find only Education^{exog.}?

Use an instrument!

Instruments

What is an instrument?

Something that is correlated with the policy variable

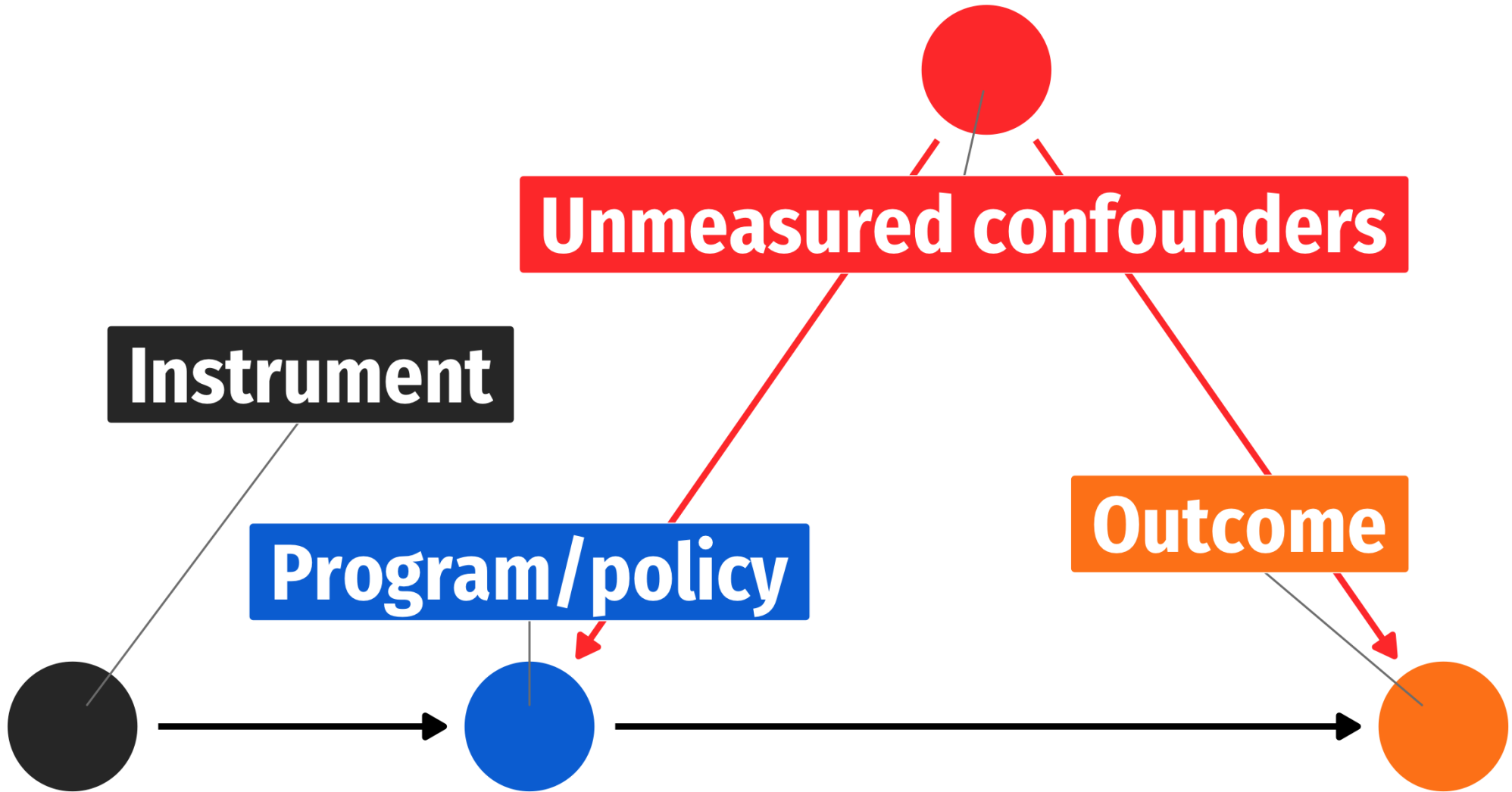
(Relevance)

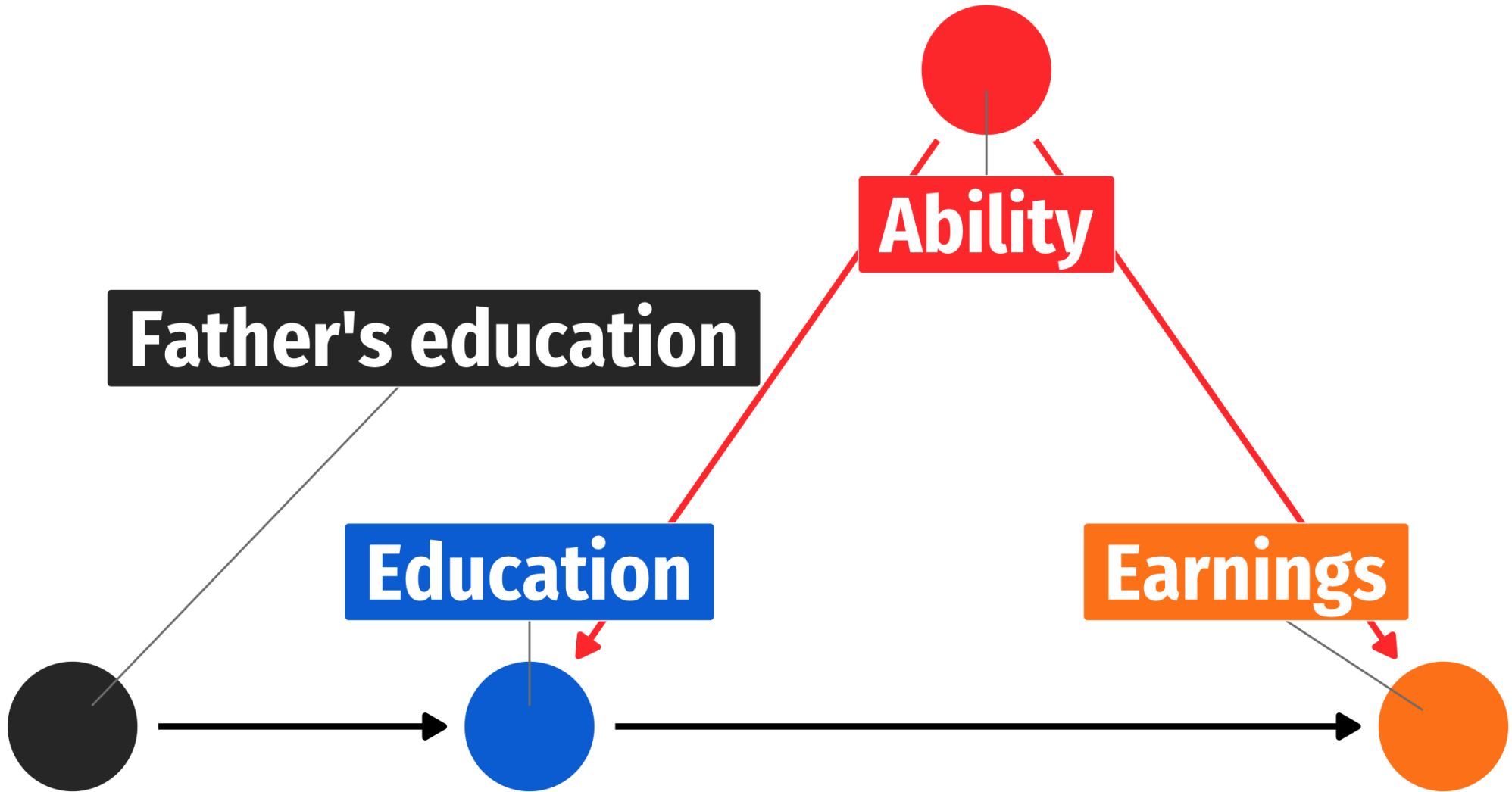
Something that does not directly cause the outcome

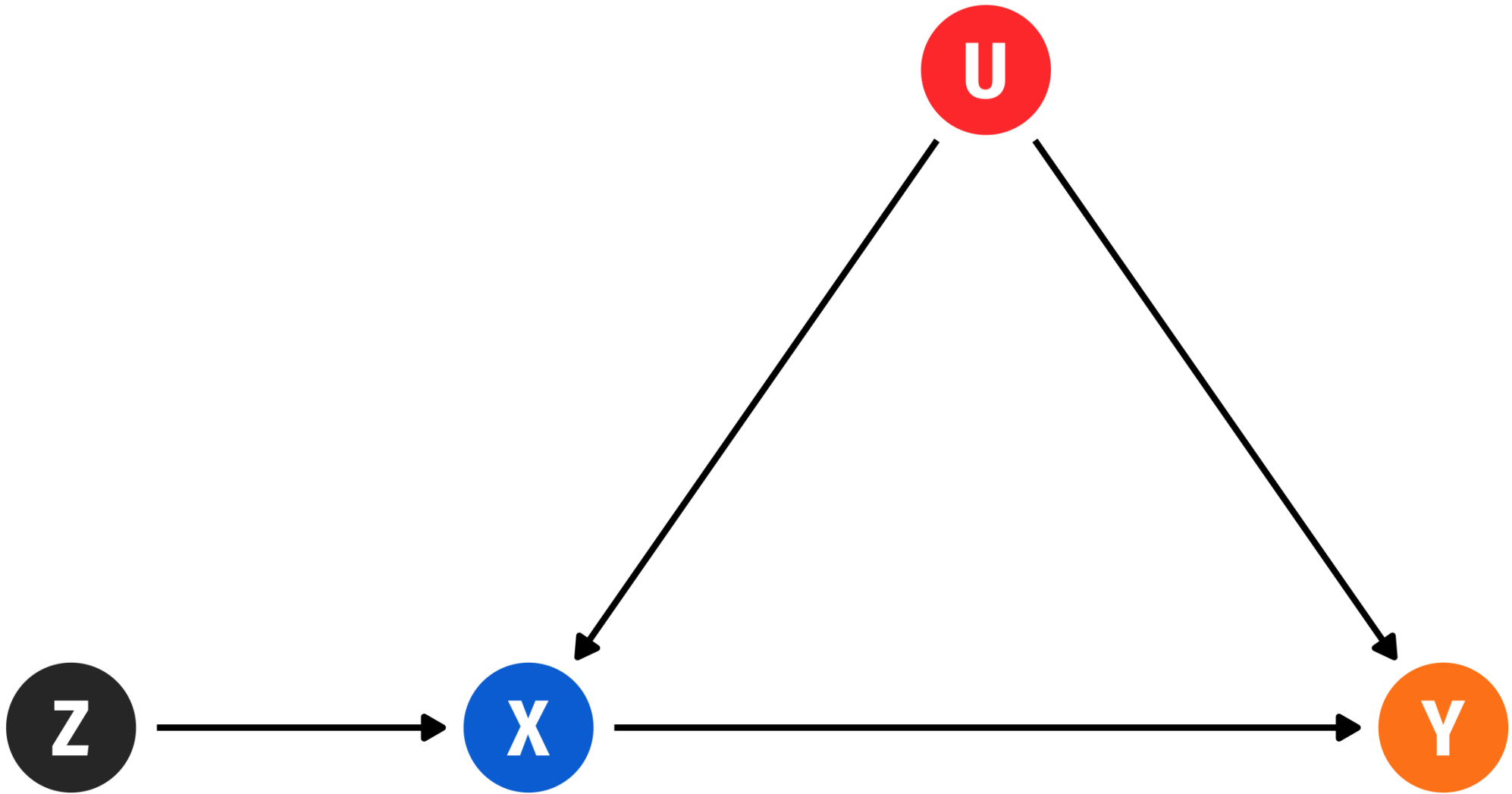
(Exclusion)

Something that is not correlated with the omitted variables

(Exogeneity)







Relevance
Correlated with policy

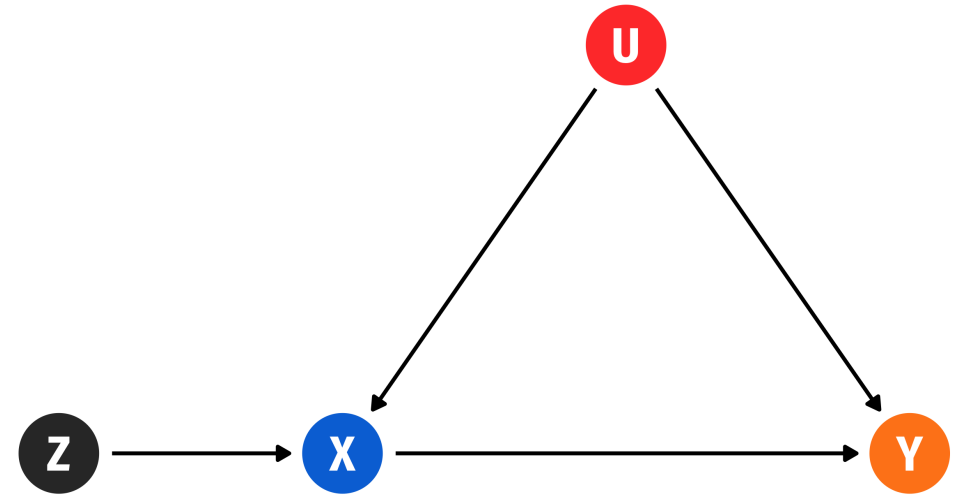
$$Z \rightarrow X \quad \text{Cor}(Z, X) \neq 0$$

Excludability
Correlated with outcome
only through policy

$$Z \rightarrow X \rightarrow Y \quad Z \not\rightarrow Y \quad \text{Cor}(Z, Y | X) = 0$$

Exogeneity
Not correlated
with omitted variables

$$U \not\rightarrow Z \quad \text{Cor}(Z, U) = 0$$



Relevance testable with stats

Excludability testable with stats + story

Exogeneity requires story, no stats

Relevance

Instrument causes change in policy

$$Z \rightarrow X \quad \text{Cor}(Z, X) \neq 0$$

Social security number

Probably not relevant (uncorrelated with education)

3rd grade test scores

Potentially relevant (early grades cause more education)

Father's education

Relevant (Educated parents cause more education)

Excludability

Instrument causes outcome *only through* policy

$$Z \rightarrow X \rightarrow Y \quad Z \not\rightarrow Y \quad \text{Cor}(Z, Y | X) = 0$$

Social security number

Exclusive (SSN isn't correlated with hourly wages)

3rd grade test scores

Potentially exclusive (early grades probably don't cause wages)

Father's education

Exclusive (Parent's education doesn't cause your wages (lol))

Exogeneity

Instrument not correlated with omitted variables

$$U \perp Z \quad \text{Cor}(Z, U) = 0$$

Social security number

Exogenous (Unrelated to anything related to education)

3rd grade test scores

Not exogenous (Grades correlated with other education factors)

Father's education

Exogenous (Birth to parents is random)

The huh? factor

"A necessary but not a sufficient condition for having an instrument that can satisfy the exclusion restriction is if people are confused when you tell them about the instrument's relationship to the outcome."

Scott Cunningham, *Causal Inference: The Mixtape*, p. 123

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|---------------|-------------------------|--------------------|
| Income | Education | Ability | Father's education |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|---------------|-------------------------|---------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|---------------|-------------------------|---------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |
| Income | Education | Ability | Military draft |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|--------------------|---------------------------------|---------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |
| Income | Education | Ability | Military draft |
| Health | Smoking cigarettes | Other negative health behaviors | Tobacco taxes |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|--------------------|---------------------------------|---------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |
| Income | Education | Ability | Military draft |
| Health | Smoking cigarettes | Other negative health behaviors | Tobacco taxes |
| Crime rate | Patrol hours | # of criminals | Election cycles |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------|--------------------|---------------------------------|--------------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |
| Income | Education | Ability | Military draft |
| Health | Smoking cigarettes | Other negative health behaviors | Tobacco taxes |
| Crime rate | Patrol hours | # of criminals | Election cycles |
| Crime | Incarceration rate | Simultaneous causality | Overcrowding litigations |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------------|--------------------|---------------------------------|--------------------------|
| Income | Education | Ability | Father's education |
| Income | Education | Ability | Distance to college |
| Income | Education | Ability | Military draft |
| Health | Smoking cigarettes | Other negative health behaviors | Tobacco taxes |
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| Labor market success | Americanization | Ability | Scrabble score of name |

| Outcome | Policy | Unobserved stuff | Instrument |
|----------------------|--------------------|---------------------------------|--------------------------|
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| Income | Education | Ability | Distance to college |
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| Crime rate | Patrol hours | # of criminals | Election cycles |
| Crime | Incarceration rate | Simultaneous causality | Overcrowding litigations |
| Labor market success | Americanization | Ability | Scrabble score of name |
| Conflicts | Economic growth | Simultaneous causality | Rainfall |

Instruments are hard to find!

The trickiest thing to prove is
the exclusion restriction

Instrument causes the outcome *only through* the policy

Most proposed instruments fail this!

Rainfall as an instrument

People love using weather as an instrument... buuuuut...

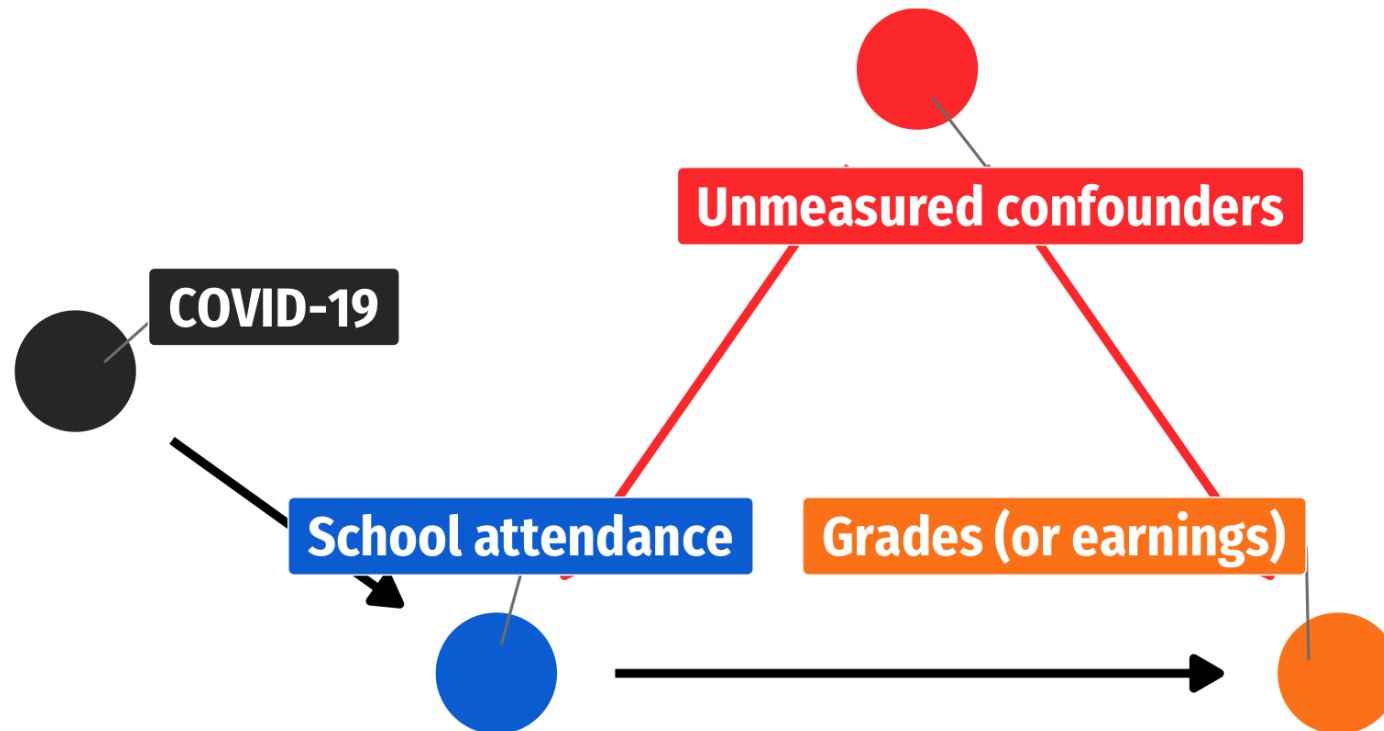
COVID-19 as an instrument

**A global pandemic is a huge
exogenous shock to
social systems everywhere**

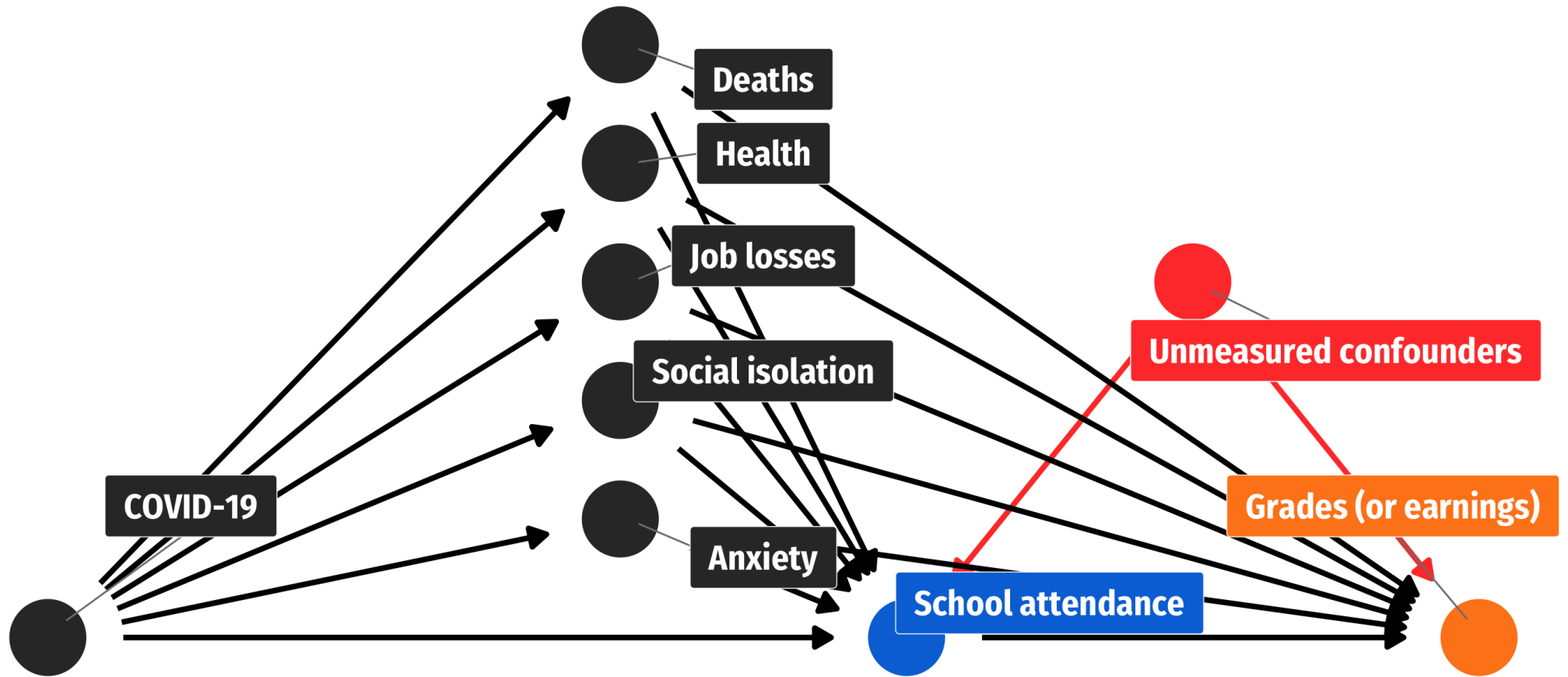
Maybe we can use it as an instrument!

COVID-19 as an instrument

What effect does closing schools have on student performance or lifetime earnings?



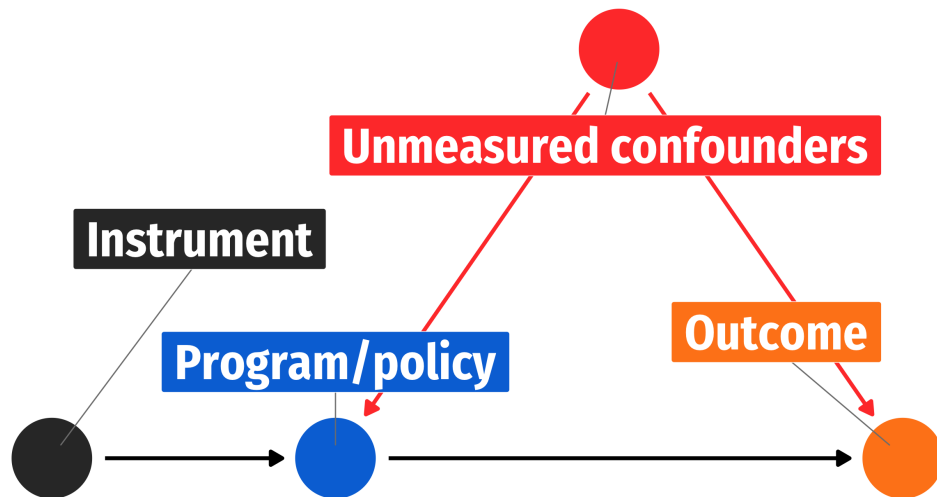
lolnope



Falsifying exclusion assumptions

Can you think of some other way that the instrument can cause the outcome outside of the policy?

If so, the instrument doesn't meet exclusion restriction



Instrument → ?? → outcome?

Rainfall → ?? → civil war?

Tobacco taxes → ?? → health?

Scrabble score → ?? →
Labor market success?

Using instruments

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

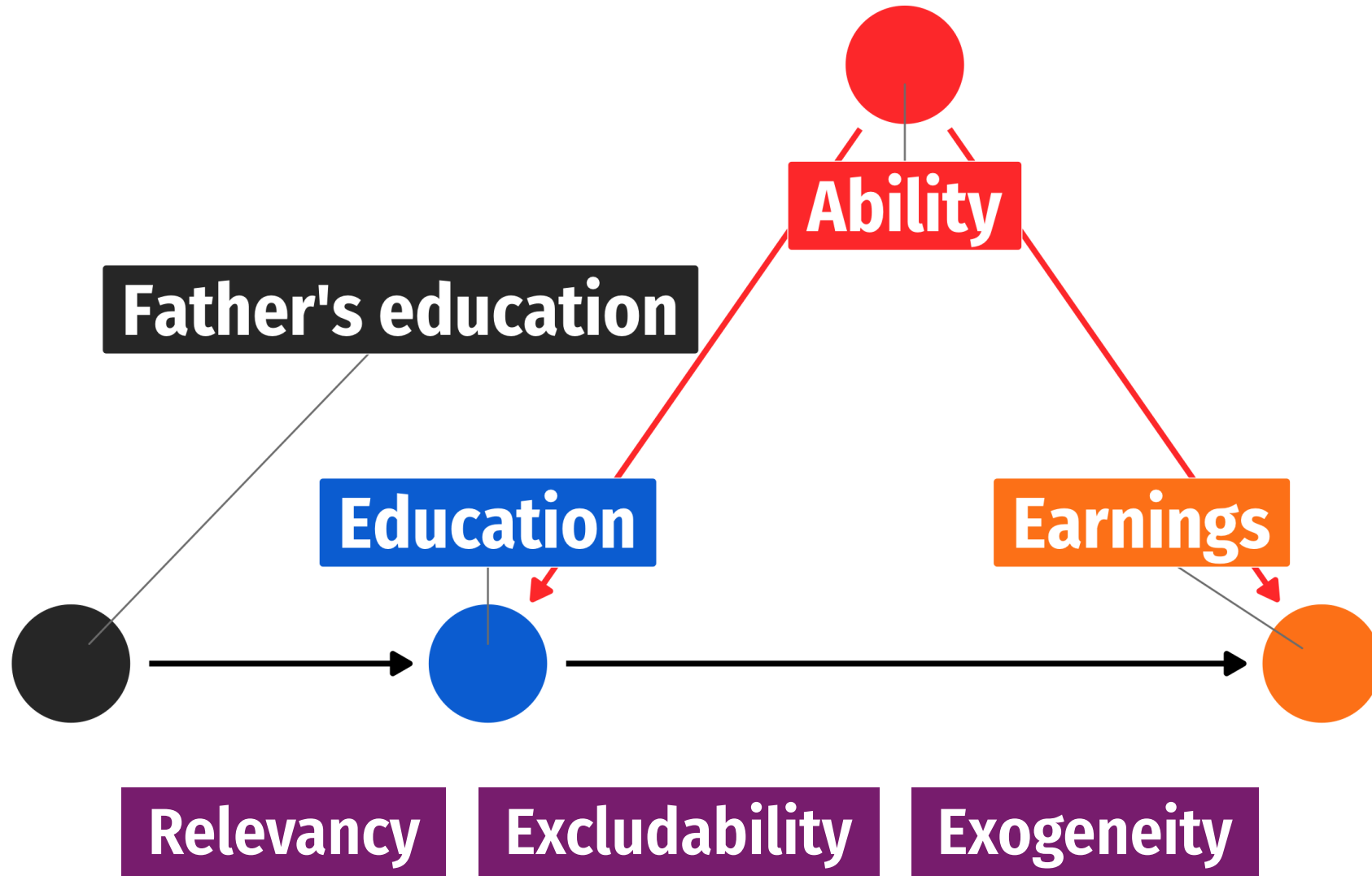
| | Unadjusted | Forbidden |
|--------------------------------------|-------------------|------------------|
| (Intercept) | -59.378*** | -85.571*** |
| | (10.376) | (7.198) |
| educ | 13.124*** | 7.767*** |
| | (0.618) | (0.456) |
| ability | | 0.344*** |
| | | (0.010) |
| Num.Obs. | 1000 | 1000 |
| R2 | 0.311 | 0.673 |
| F | 451.244 | 1025.794 |
| * p < 0.1, ** p < 0.05, *** p < 0.01 | | |

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

$$\beta_0 + \beta_1 (\text{Education}_i^{\text{exog.}} + \text{Education}_i^{\text{endog.}}) + \varepsilon_i$$

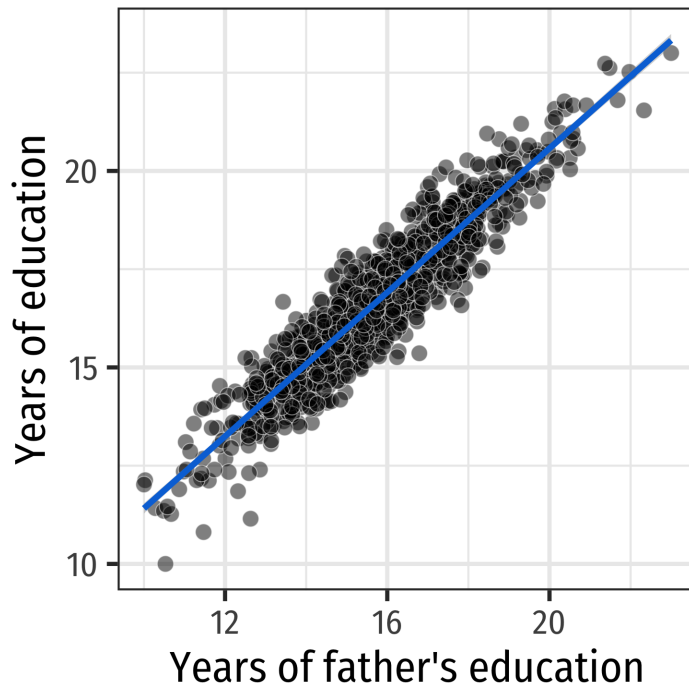
$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \underbrace{\beta_1 \text{Education}_i^{\text{endog.}}}_{\omega_i} + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$



Relevancy

Program ~ instrument



Clear, significant effect = relevant!

```
first_stage <- lm(educ ~ fathereduc, data = father_education)
tidy(first_stage)
```

```
## # A tibble: 2 x 5
##   term      estimate std.error statistic  p.value
##   <chr>      <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept)  2.25     0.172     13.1 3.67e-36
## 2 fathereduc  0.916    0.0108    84.5 0.
```

First-stage model F-statistic (statistic here) > 104 = strong instrument

```
glance(first_stage)
```

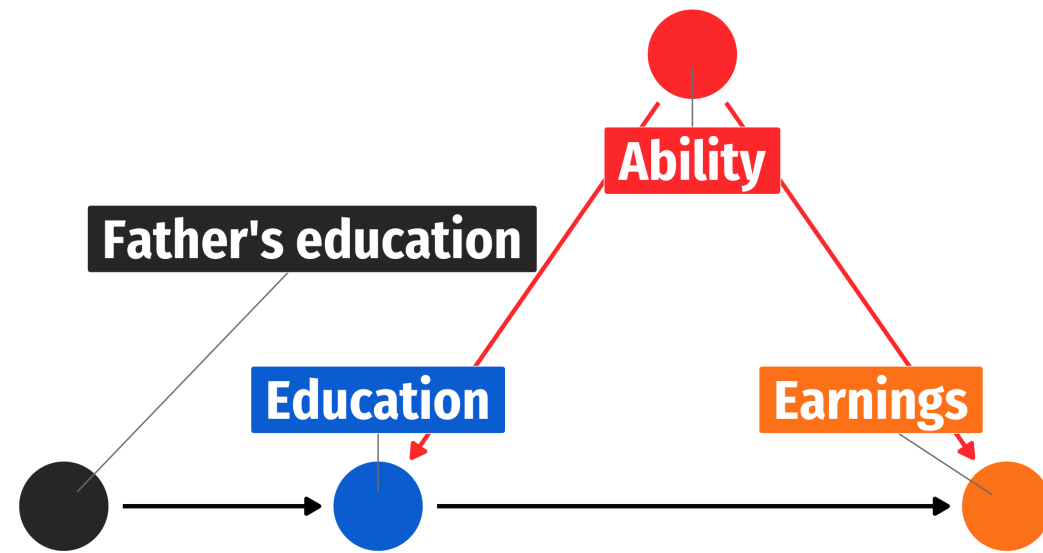
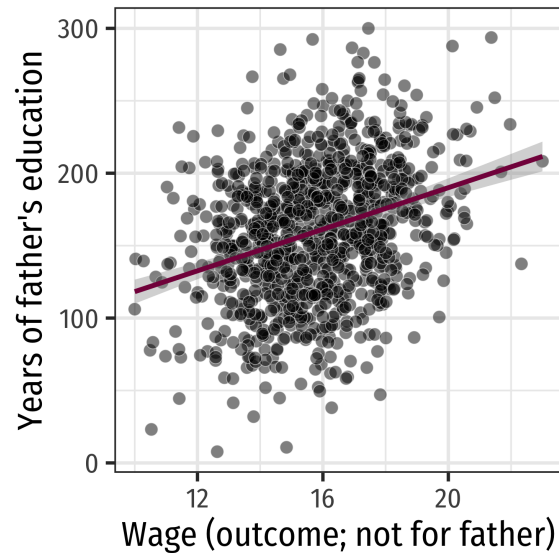
```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic p.value  df logLik  AIC  BIC
##   <dbl>      <dbl> <dbl>    <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl>
## 1  0.877      0.877 0.703    7136.     0     1 -1066. 2137. 2152.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Exclusion

Does it meet exclusion assumption?

Father's education causes your wages *only through* your education?

Any other plausible node between father's education and earnings?



Exogeneity

Is assignment to your parents random?

Sure.

Is your parents' choice to gain education random?

lolz.

Two-stage least squares (2SLS)

Find exogenous part of policy variable based on instrument; use *that* to predict outcome

First stage

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + v_i$$

Second stage

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

"Education hat": fitted/predicted values;
exogenous part of education

Stage 1: Policy ~ instrument

```
first_stage <- lm(educ ~ fathereduc, data = father_education)
tidy(first_stage)
```

```
## # A tibble: 2 x 5
##   term          estimate std.error statistic  p.value
##   <chr>         <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)    2.25      0.172      13.1 3.67e-36
## 2 fathereduc    0.916     0.0108     84.5 0.
```

Stage 1: Check instrument strength

Model's F-statistic (*statistic here*) **should be > 104**
(though most books say > 10)

```
glance(first_stage)
```

```
## # A tibble: 1 x 5
##   r.squared adj.r.squared sigma statistic p.value
##   <dbl>      <dbl> <dbl>      <dbl>    <dbl>
## 1     0.877     0.877  0.703     7136.      0
```

Stage 1: Use first stage to predict policy

$$\widehat{\text{Education}}_i = 2.251 + (0.916 \times \text{Father's education}_i) + v_i$$

```
data_with_predictions <- augment_columns(first_stage, data = father_education) %>%  
  rename(educ_hat = .fitted)  
head(data_with_predictions)
```

```
## # A tibble: 6 x 5  
##   wage educ ability fathereduc educ_hat  
##   <dbl> <dbl> <dbl> <dbl> <dbl>  
## 1  180.  18.5  408.   17.2  18.0  
## 2  100.  16.2  310.   15.5  16.4  
## 3  125.  18.2  303.   17.7  18.4  
## 4  178.  16.6  342.   15.6  16.5  
## 5  265.  17.3  534.   14.7  15.8  
## 6  187.  17.5  409.   16.0  16.9
```

$$\text{educ_hat} = 2.251 + (0.916 \times 17.2) = 18.0$$

$$\text{educ_hat} = 2.251 + (0.916 \times 15.5) = 16.4$$

Stage 2: Outcome ~ predicted policy

```
second_stage <- lm(wage ~ educ_hat,  
                  data = data_with_predictions)
```

```
tidy(second_stage)
```

```
## # A tibble: 2 x 5
```

```
##   term          estimate std.error statistic  p.value  
##   <chr>         <dbl>     <dbl>     <dbl>    <dbl>  
## 1 (Intercept)    28.8      12.7       2.27 2.32e- 2  
## 2 educ_hat       7.83      0.755     10.4 5.10e-24
```

| | Unadjusted | Forbidden | 2SLS IV |
|--------------------------------------|-------------------|------------------|----------------|
| (Intercept) | -59.378*** | -85.571*** | 28.819** |
| | (10.376) | (7.198) | (12.672) |
| educ | 13.124*** | 7.767*** | |
| | (0.618) | (0.456) | |
| ability | | 0.344*** | |
| | | (0.010) | |
| educ_hat | | | 7.835*** |
| | | | (0.755) |
| Num.Obs. | 1000 | 1000 | 1000 |
| R2 | 0.311 | 0.673 | 0.097 |
| F | 451.244 | 1025.794 | 107.639 |
| * p < 0.1, ** p < 0.05, *** p < 0.01 | | | |

**Unadjusted
is wrong!**

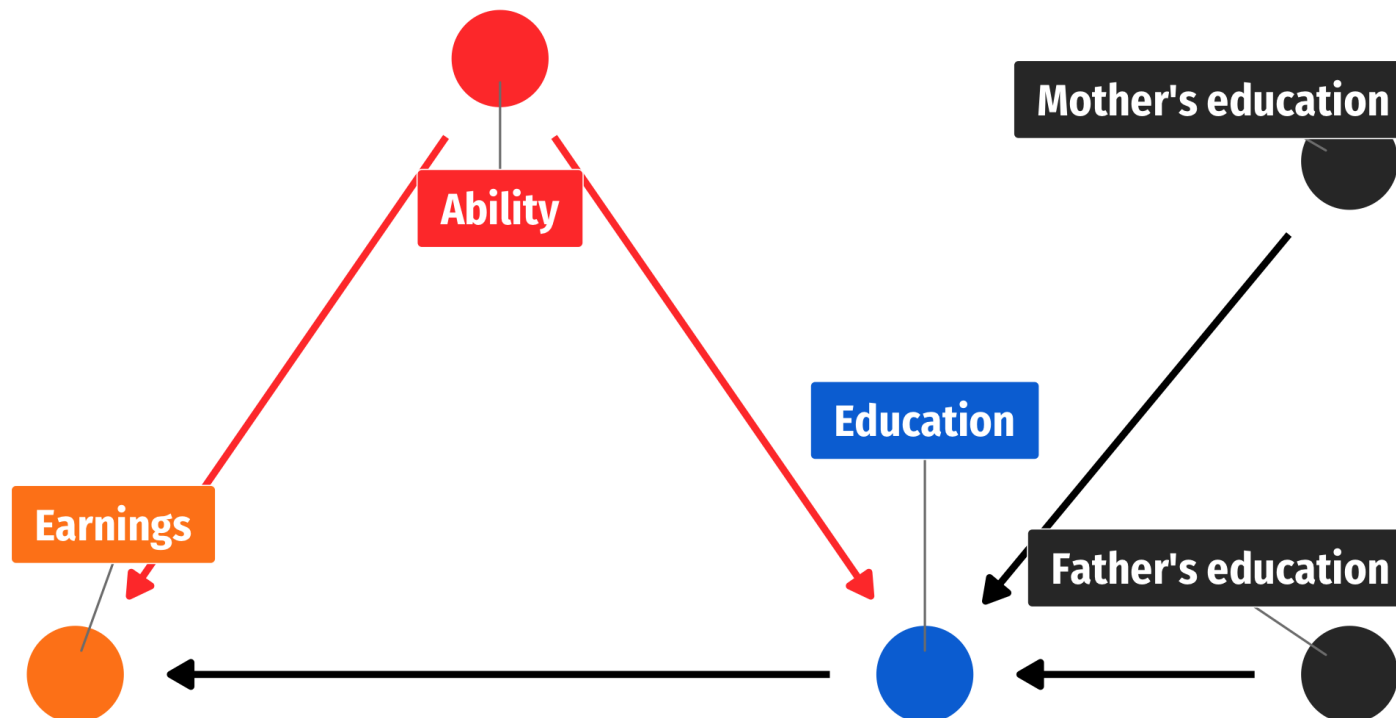
**Forbidden is right,
but not actually
measurable!**

**2SLS is close
and measurable!**

**One year of education
causes hourly wage to
increase by \$7.84**

Multiple instruments

You can use multiple instruments to explain more of the endogeneity in the policy node



Multiple instruments

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + \gamma_2 \text{Mother's education}_i + v_i$$

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

Other control variables

You can use control variables too!

For mathy reasons,
all exogenous controls need to go in both stages

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + \gamma_2 \text{Mother's education}_i + \gamma_3 \text{SES}_i + \gamma_4 \text{State}_i + \gamma_5 \text{Year}_i + v_i$$

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \beta_2 \text{SES}_i + \beta_3 \text{State}_i + \beta_4 \text{Year}_i + \varepsilon_i$$

Faster, more accurate ways to run 2SLS

Running the first stage, calculating policy-hat, then running second stage is neat, but time consuming!

```
first_stage <- lm(educ ~ fathereduc, data = father_education)
data_with_predictions <- augment_columns(first_stage, data = father_education) %>%
  rename(educ_hat = .fitted)
second_stage <- lm(wage ~ educ_hat, data = data_with_predictions)
```

Your standard errors will be wrong unless you adjust them with fancy math by hand

Use R packages that do all that work for you instead!

Faster, more accurate ways to run 2SLS

`ivreg()` from the `ivreg` package

Outcome ~ 2nd stage stuff | 1st stage stuff

```
library(ivreg)
model_ivreg <- ivreg(wage ~ educ | fathereduc,
                    data = father_education)
tidy(model_ivreg)
```

```
## # A tibble: 2 x 5
##   term      estimate std.error statistic  p.value
##   <chr>      <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept) 28.8      11.5      2.51 1.21e- 2
## 2 educ        7.83     0.683    11.5 1.13e-28
```

```
summary(model_ivreg)
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 28.8187    11.4679   2.513  0.0121 *
## educ        7.8349     0.6834  11.465 <2e-16 ***
##
## Diagnostic tests:
##              df1 df2  statistic p-value
## Weak instruments  1 998    7136 <2e-16 ***
## Wu-Hausman       1 997    1102 <2e-16 ***
```

Faster, more accurate ways to run 2SLS

`iv_robust()` from the **estimatr** package

Outcome ~ 2nd stage stuff | 1st stage stuff

```
library(estimatr)
model_iv_robust <- iv_robust(wage ~ educ | fathereduc,
                             data = father_education)
tidy(model_iv_robust)
```

```
##           term estimate std.error statistic      p.value conf.low conf.high
## 1 (Intercept) 28.818695 11.1645893  2.581259 9.985789e-03 6.909932 50.727459
## 2          educ  7.834935  0.6635423 11.807739 3.281862e-30 6.532837  9.137033
##    df outcome
## 1 998   wage
## 2 998   wage
```

(See also `rfc()` from the **felm** package for IV with fancy fixed effects)

| | Unadjusted | Forbidden | 2SLS IV (by hand) | 2SLS IV (ivreg()) | 2SLS IV (iv_robust()) |
|--------------------------------------|-------------------|------------------|--------------------------|--------------------------|------------------------------|
| (Intercept) | -59.378*** | -85.571*** | 28.819** | 28.819** | 28.819** |
| | (10.376) | (7.198) | (12.672) | (11.468) | (11.165) |
| educ | 13.124*** | 7.767*** | | 7.835*** | 7.835*** |
| | (0.618) | (0.456) | | (0.683) | (0.664) |
| ability | | 0.344*** | | | |
| | | (0.010) | | | |
| educ_hat | | | 7.835*** | | |
| | | | (0.755) | | |
| Num.Obs. | 1000 | 1000 | 1000 | 1000 | |
| R2 | 0.311 | 0.673 | 0.097 | 0.261 | 0.261 |
| R2 Adj. | 0.311 | 0.672 | 0.096 | 0.260 | 0.260 |
| * p < 0.1, ** p < 0.05, *** p < 0.01 | | | | | |

General IV process

1: Is the instrument relevant?

Instrument correlated with policy/program; F-statistic in 1st stage > 104

2: Does the instrument meet exclusion assumption?

Instrument causes outcome *only through* policy/program. **Good luck.**

3: Is the instrument exogenous?

No arrows going into instrument node in DAG

4: 2-stage least squares (2SLS)

`program ~ instrument; outcome ~ program_hat` **OR** `iv_robust()`